

Pearson International GCSE in Mathematics (Specification A) (4MA1)

Two-year Scheme of Work

For first teaching from September 2016

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Introduction

This scheme of work is based upon a five-term model over two years for both Foundation and Higher tier students. It can be used directly as a scheme of work for the International GCSE Mathematics (Specification A) (4MA1).

The scheme of work is broken up into two tiers, and then into units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Tier
- Contents, referenced back to the specification
- Prior knowledge
- Keywords.

Each sub-unit contains:

- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Possible success criteria for students at the end of the sub-unit
- Opportunities for reasoning/problem-solving
- Common misconceptions
- Notes for general mathematical teaching points.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

International GCSE Mathematics

(Specification A)

Foundation Tier

Scheme of Work

Unit number		Title	Estimated teaching hours
Number	1	Integers and place value	4
	2	Decimals	4
	3	Special numbers and powers	7
	4	Fractions	4
	5	Percentages	9
	6	Ratio and proportion	7
	7	Arithmetic of fractions	4
	8	Set language, notation and Venn diagrams	7
	9	Indices and standard form	5
Algebra	10	Algebraic manipulation	5
	11	Expressions, formulae and rearranging formulae	6
	12	Linear equations and inequalities	8
	13	Sequences	5
	14	Real life graphs	4
	15	Linear graphs	6
	16	Quadratic equations and graphs	5
	17	Simultaneous equations	4
Space, shape and measure	18	Measures, bearings and scale drawings	5
	19	Symmetry, shapes, parallel lines and angle facts	8
	20	Interior and exterior angles of polygons	5
	21	Compound measures	5
	22	Perimeter, area and volume	6
	23	Circles and cylinders	6
	24	Transformations	7
	25	Pythagoras' theorem and Trigonometry	12
	26	Similarity and congruence in 2D	5
	27	Constructions	4
Handling Data	28	Graphical representation of data	7
	29	Statistical measures	7
	30	Probability	9
		Total	180

Unit	Title	Specification Reference	Estimated teaching hours
5	Percentages	1.6A understand that 'percentage' means 'number of parts per 100'	9
		1.6B express a given number as a percentage of another number	
		1.6C express a percentage as a fraction and as a decimal	
		1.6D understand the multiplicative nature of percentages as operators	
		1.6E solve simple percentage problems, including percentage increase and decrease	
		1.6F use reverse percentages	
		1.6G use compound interest and depreciation	
6	Ratio and proportion	1.7A use ratio notation, including reduction to its simplest form and its various links to fraction notation	7
		1.7B divide a quantity in a given ratio or ratios	
		1.7C use the process of proportionality to evaluate unknown quantities	
		1.7D calculate an unknown quantity from quantities that vary in direct proportion	
		1.7E solve word problems about ratio and proportion	
		1.10A use and apply number in everyday personal, domestic or community life	
		1.10B carry out calculations using standard units of mass, length, area, volume and capacity	
		1.10C understand and carry out calculations using time, and carry out calculations using money, including converting between currencies	
7	Arithmetic of fractions	1.2F use common denominators to add and subtract fractions and mixed numbers	4
		1.2H understand and use fractions as multiplicative inverses	
		1.2I multiply and divide fractions and mixed numbers	
8	Set language, notation and Venn diagrams	1.5A understand the definition of a set	7
		1.5B use the set notation \cup , \cap and \in and \notin	
		1.5C understand the concept of the universal set and the empty set and the symbols for these sets	
		1.5D understand and use the complement of a set	
		1.5E use Venn diagrams to represent sets	
		6.3D find probabilities from a Venn diagram	
9	Indices and standard	1.4C use index notation and index laws for multiplication and division of positive and negative integer powers including zero	5

	form	1.9A calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$	
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Algebra : Units 10 – 17

OBJECTIVES / SPECIFICATION REFERENCES

Unit	Title	Specification Reference	Estimated teaching hours
10	Algebraic manipulation	2.1A understand that symbols may be used to represent numbers in equations or variables in expressions and formulae	5
		2.1B understand that algebraic expressions follow the generalised rules of arithmetic	
		2.1C use index notation for positive and negative integer powers (including zero)	
		2.1D use index laws in simple cases	
		2.2B collect like terms	
		2.2C multiply a single term over a bracket	
		2.2D take out common factors	
11	Expressions, formulae and rearranging formulae	2.2A evaluate expressions by substituting numerical values for letters	6
		2.3A understand that a letter may represent an unknown number or a variable	
		2.3B use correct notational conventions for algebraic expressions and formulae	
		2.3C substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae	
		2.3D use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols	
		2.3E derive a formula or expression	
		2.3F change the subject of a formula where the subject appears once	
12	Linear equations and inequalities	2.4A solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation	8
		2.4B set up simple linear equations from given data	
		2.8A understand and use the symbols $>$, $<$, \leq and \geq	
		2.8B understand and use the convention for open and closed intervals on a number line	

		2.8C solve simple linear inequalities in one variable and represent the solution set on a number line	
Unit	Title	Specification Reference	Estimated teaching hours
13	Sequences	3.1A generate terms of a sequence using term-to-term and position-to-term definitions of the sequence	5
		3.1B find subsequent terms of an integer sequence and the rule for generating it	
		3.1C use linear expressions to describe the n th term of arithmetic sequences	
14	Real life graphs	3.3A interpret information presented in a range of linear and non-linear graphs	4
15	Linear graphs	3.3B understand and use conventions for rectangular Cartesian coordinates	6
		3.3C plot points (x, y) in any of the four quadrants or locate points with given coordinates	
		3.3D determine the coordinates of points identified by geometrical information	
		3.3E determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points	
		3.3F draw and interpret straight line conversion graphs	
		3.3G find the gradient of a straight line	
		3.3H recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$	
		3.3I recognise, generate points and plot graphs of linear functions	
		2.8D represent simple linear inequalities on rectangular Cartesian graphs	
16	Quadratic equations and graphs	2.2E expand the product of two simple linear expressions	5
		2.2F understand the concept of a quadratic expression and be able to factorise such expressions (limited to $x^2 + bx + c$)	
		2.7A solve quadratic equations by factorization (limited to $x^2 + bx + c = 0$)	
		3.3I recognise, generate points and plot graphs quadratic functions	
17	Simultaneous equations	2.6A calculate the exact solution of two simultaneous equations in two unknowns	4

Shape, space and measure : Units 18 - 27

OBJECTIVES / SPECIFICATION REFERENCES

Unit	Title	Specification Reference	Estimated teaching hours
18	Measures, bearings and scale drawings	4.4A interpret scales on a range of measuring instruments	5
		4.4B calculate time intervals in terms of the 24-hour and the 12-hour clock	
		4.4C make sensible estimates of a range of measures	
		4.4D understand angle measure including three-figure bearings	
		4.1A distinguish between acute, obtuse, reflex and right angles	
		4.4E measure an angle to the nearest degree	
		4.5A measure and draw lines to the nearest millimetre	
		4.5C solve problems using scale drawings	
		4.11B use and interpret maps and scale drawings	
		4.9A convert measurements within the metric system to include linear and area units	
		4.10A convert between units of volume within the metric system	
19	Symmetry, shapes, parallel lines and angle facts	4.3A identify any lines of symmetry and the order of rotational symmetry of a given two-dimensional figure	8
		4.1B use angle properties of intersecting lines, parallel lines and angles on a straight line	
		4.1C understand the exterior angle of a triangle property and the angle sum of a triangle property	
		4.1D understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles	
		4.2B understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals	
		4.2C understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite	
		4.7A give informal reasons, where required, when arriving at numerical solutions to geometrical problems	
		4.10A recognise and give the names of solids	
		4.10B understand the terms 'face', 'edge' and 'vertex' in the context of 3-D solids	

Unit	Title	Specification Reference	Estimated teaching hours
20	Polygons	4.2A recognise and give the names of polygons	5
		4.2D understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons	
		4.2E understand and use the angle sum of polygons	
21	Compound measures	4.4F understand and use the relationship between average speed, distance and time	5
		4.4G use compound measure such as speed, density and pressure	
22	Perimeter, area and volume	4.9B find the perimeter of shapes made from triangles and rectangles	6
		4.9C find the area of simple shapes using the formulae for the areas of triangles and rectangles	
		4.9D find the area of parallelograms and trapezia	
		4.10C find the surface area of simple shapes using the area formulae for triangles and rectangles	
		4.10E find the volume of prisms, including cuboids and cylinders, using an appropriate formula	
23	Circles and cylinders	4.6A recognise the terms 'centre', 'radius', 'chord', 'diameter', 'circumference', 'tangent', 'arc', 'sector' and 'segment' of a circle	6
		4.6B understand chord and tangent properties of circles	
		4.9E find circumferences and areas of circles using relevant formulae; find perimeters and areas of semicircles	
		4.10D find the surface area of a cylinder	
		4.10E find the volume of prisms, including cuboids and cylinders, using an appropriate formula	

Unit	Title	Specification Reference	Estimated teaching hours
24	Transformations	5.2A understand that rotations are specified by a centre and an angle	7
		5.2B rotate a shape about a point through a given angle	
		5.2C recognise that an anti-clockwise rotation is a <i>positive</i> angle of rotation and a clockwise rotation is a <i>negative</i> angle of rotation	
		5.2D understand that reflections are specified by a mirror line	
		5.2E construct a mirror line given an object and reflect a shape given a mirror line	
		5.2F understand that translations are specified by a distance and direction	
		5.2G translate a shape	
		5.2H understand and use column vectors in translations	
		5.2I understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape	
		5.2J understand that enlargements are specified by a centre and a scale factor	
		5.2K understand that enlargements preserve angles and not lengths	
		5.2L enlarge a shape given the scale factor	
		5.2M identify and give complete descriptions of transformations	
25	Pythagoras' theorem and Trigonometry	4.8A know, understand and use Pythagoras' Theorem in two dimensions	12
		4.8B know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle	
		4.8C apply trigonometrical methods to solve problems in two dimensions	
26	Similarity and congruence in 2D	4.2F understand congruence as meaning the same shape and size	5
		4.2G understand that two or more polygons with the same shape and size are said to be congruent to each other	
		4.11A understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged	
27	Constructions and bearings	4.5B construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses	4
		4.5D use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle	

Handling Data : Units 28 – 30

OBJECTIVES / SPECIFICATION REFERENCES

Unit	Title	Specification Reference	Estimated teaching hours
28	Graphical representation of data	6.1A use different methods of presenting data	7
		6.1B use appropriate methods of tabulation to enable the construction of statistical diagrams	
		6.1C interpret statistical diagrams	
29	Statistical measures	6.2A understand the concept of average	7
		6.2B calculate the mean, median, mode and range for a discrete data set	
		6.2C calculate an estimate for the mean for grouped data	
		6.2D identify the modal class for grouped data	
30	Probability	6.3A understand the language of probability	9
		6.3B understand and use the probability scale	
		6.3C understand and use estimates or measures of probability from theoretical models	
		6.3D find probabilities from a Venn diagram	
		6.3E understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space	
		6.3F list all the outcomes for single events and for two successive events in a systematic way	
		6.3G estimate probabilities from previously collected data	
		6.3H calculate the probability of the complement of an event happening	
		6.3I use the addition rule of probability for mutually exclusive events	
		6.3J understand and use the term 'expected frequency'	

4. Fractions**Teaching time**

3 - 5 hours

OBJECTIVES

1.2A	understand and use equivalent fractions, simplifying a fraction by cancelling common factors
1.2B	understand and use mixed numbers and vulgar fractions
1.2C	identify common denominators
1.2D	order fractions and calculate a given fraction of a given quantity
1.2E	express a given number as a fraction of another number
1.2G	convert a fraction to a decimal or percentage

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction > 1 .

Simplify $\frac{120}{100}$.

Find $\frac{3}{5}$ of 15, $\frac{3}{4}$ of 20.

Find $\frac{1}{2}$ of 36 m, $\frac{1}{4}$ of £20.

Find the size of each category from a pie chart using fractions.

Write $\frac{2}{5}$ as (i) a decimal (ii) a percentage

Write $\frac{52}{6}$ as a mixed number in its simplest form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1 , e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.

Working out the number of people/things where the number of people/things in different categories is given as a fraction.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

NOTES

When expressing a given number as a fraction of another, start with very simple numbers < 1 , and include some cancelling before fractions using numbers > 1 .

Regular revision of fractions is essential.

Demonstrate how to use the fraction button on the calculator.

Use real-life examples where possible.

EXAMPLE QUESTIONS FROM SAMs: 1F q2

5. Percentages**Teaching time**

8 - 10 hours

OBJECTIVES

1.6A	understand that 'percentage' means 'number of parts per 100'
1.6B	express a given number as a percentage of another number
1.6C	express a percentage as a fraction and as a decimal
1.6D	understand the multiplicative nature of percentages as operators
1.6E	solve simple percentage problems, including percentage increase and decrease
1.6F	use reverse percentages
1.6G	use compound interest and depreciation

POSSIBLE SUCCESS CRITERIA

What is 10%, 15%, 17.5% of £30?

Write 64% as (i) a decimal (ii) as a fraction in its simplest form.

Jan's salary is £24 000. She gets a pay rise of 6%, work out her new salary.

Find the total interest is £4500 is invested for 3 years at 2.5% compound interest.

Normal prices are reduced by 15% in a sale. Find the normal price of an item with sale price £55.42

A car is bought for £2300 and sold for £4000. Find the percentage profit.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Sale prices offer an ideal opportunity for solving problems allowing students the opportunity to investigate the most effective way to work out the "sale" price.

Problems that involve consecutive reductions such as: Sale Prices are 10% off the previous day's price. If a jacket is £90 on Monday, what is the price on Wednesday?

COMMON MISCONCEPTIONS

It is not possible to have a percentage greater than 100%.

NOTES

Amounts of money should always be rounded to two decimal places.

Use real-life examples where possible.

Emphasise the importance of being able to convert between decimals and percentages and the use of decimal multipliers to make calculations easier.

EXAMPLE QUESTIONS FROM SAMs: 1F q2, q19, q23; 2F q20, q23

6. Ratio and proportion**Teaching time**

8 - 10 hours

OBJECTIVES

1.7A	use ratio notation, including reduction to its simplest form and its various links to fraction notation
1.7B	divide a quantity in a given ratio or ratios
1.7C	use the process of proportionality to evaluate unknown quantities
1.7D	calculate an unknown quantity from quantities that vary in direct proportion
1.7E	solve word problems about ratio and proportion
1.10A	use and apply number in everyday personal, domestic or community life
1.10B	carry out calculations using standard units of mass, length, area, volume and capacity
1.10C	understand and carry out calculations using time, and carry out calculations using money, including converting between currencies

POSSIBLE SUCCESS CRITERIA

Write a ratio to describe a situation such as 1 blue for every 2 red, or 3 adults for every 10 children.

Share \$98 in the ratio 2 : 3 : 5

If £1 = \$1.42, how many \$ do you get for £50; how many £ do you get for \$67 ?

Scale up recipes and decide if there is enough of each ingredient.

Given two sets of data in a table, are they in direct proportion?

A film starts at 11:50 and ends at 13:35, how long did it last?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Anna, Bob and Clive share some money in the ratio 1 : 2 : 4. Clive gets £36 more than Anna. How much did Bob get?

Problems in context, such as scaling a recipe, or diluting lemonade or chemical solutions, will show how proportional reasoning is used in real-life contexts.

COMMON MISCONCEPTIONS

Using a ratio to find one quantity when the other is known often results in students 'sharing' the known amount.

NOTES

Emphasise the importance of reading the question carefully.

Include ratios with decimals 0.2 : 1.

Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values.

EXAMPLE QUESTIONS FROM SAMs: 1F q15, q17; 2F q10, q15

7. Arithmetic of fractions**Teaching time**

3 - 5 hours

OBJECTIVES

1.2F	use common denominators to add and subtract fractions and mixed numbers
1.2H	understand and use fractions as multiplicative inverses
1.2I	multiply and divide fractions and mixed numbers

POSSIBLE SUCCESS CRITERIA

$$\frac{3}{5} \times 15, 20 \times \frac{3}{4}.$$

$$\frac{1}{2} \text{ of } 36 \text{ m}, \frac{1}{4} \text{ of } £20.$$

$$\text{Calculate: } \frac{1}{2} \times \frac{6}{7}, \frac{3}{5} \div 3.$$

$$\text{Work out } 2\frac{2}{3} + 1\frac{4}{5}; 2\frac{2}{3} - 1\frac{4}{5}; 2\frac{2}{3} \times 1\frac{4}{5}; 2\frac{2}{3} \div 1\frac{4}{5}$$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1 , e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.

Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

You add fractions by adding the numerators and then the denominators.

NOTES

When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers ≤ 1), and finally where both denominators have to be changed (answers ≤ 1).

Regular revision of fractions is essential.

Demonstrate how to use the fraction button on the calculator.

Use real-life examples where possible.

EXAMPLE QUESTIONS FROM SAMs: 2F q25

8. Set language, notation and Venn diagrams**Teaching time**

6 - 8 hours

OBJECTIVES

1.5A	understand the definition of a set
1.5B	use the set notation \cup , \cap and \in and \notin
1.5C	understand the concept of the universal set and the empty set and the symbols for these sets
1.5D	understand and use the complement of a set
1.5E	use Venn diagrams to represent sets
6.3D	find probabilities from a Venn diagram

POSSIBLE SUCCESS CRITERIA

Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$; Write down $A \cap B$, $A \cup B$

$C = \{1, 3, 5\}$; write down C'

Is $4 \in C$, is $4 \in A$

Draw a Venn diagram to show the universal set, A and B ;

If a number is picked at random, find $P(A \cap B)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$

Write down a possible set C so that $A \cap C = \{7\}$ and C has 4 members

COMMON MISCONCEPTIONS

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 3, 5, 5, 7, 7, 9\}$

NOTES

When drawing a Venn diagram it is a good idea to put members in the intersection first.

EXAMPLE QUESTIONS FROM SAMs: -

9. Indices and standard form**Teaching time**

4-6 hours

OBJECTIVES

1.4C	use index notation and index laws for multiplication and division of positive and negative integer powers including zero
1.9A	calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$

POSSIBLE SUCCESS CRITERIA

Write 51 080 in standard form.

Write 3.74×10^{-6} as an ordinary number.

What is 9^0 ?

Simplify $6^9 \times 6^{13}$; $4^{12} \div 4^2$;

Evaluate $(2^{-3} \times 2^5) \div 2^4$.

Write, as a single power of 7, $7^{13} \times 7^5$

Work out $(1.2 \times 10^4) \times (3 \times 10^{-9})$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link with other areas of mathematics, such as compound measures, by using speed of light in standard form.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.

NOTES

Standard form is used in science and there are lots of cross curricular opportunities.

Students need to be provided with plenty of practice in using standard form with calculators.

EXAMPLE QUESTIONS FROM SAMs: 1F q24

10. Algebraic manipulation**Teaching time**

4-6 hours

OBJECTIVES

2.1A	understand that symbols may be used to represent numbers in equations or variables in expressions and formulae
2.1B	understand that algebraic expressions follow the generalised rules of arithmetic
2.1C	use index notation for positive and negative integer powers (including zero)
2.1D	use index laws in simple cases
2.2B	collect like terms
2.2C	multiply a single term over a bracket
2.2D	take out common factors

POSSIBLE SUCCESS CRITERIASimplify $4p - 2q + 3p + 5q$ Simplify $5(a + 2b) - 3(3a - b)$ Expand $5(2x + 3)$; $x(x + 2)$ Factorise $18a + 27$; $a^2 + 3a$; $12m^3 + 9m^2$ Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2 p^0$ Simplify $x^{-4} \times x^2$, $w^2 \div w^{-1}$.**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Forming expressions and equations using area and perimeter of 2D shapes.

COMMON MISCONCEPTIONS

Any poor number skills involving negatives and times tables will become evident.

A common misconception is $3(x + 4) = 3x + 4$.The convention of not writing a coefficient with a single value, i.e. x instead of $1x$, may cause confusion.**NOTES**Emphasise correct use of symbolic notation, i.e. $3 \times y = 3y$ and not $y3$ and $a \times b = ab$.

Use lots of concrete examples when writing expressions, e.g. 'B' boys + 'G' girls.

Plenty of practice should be given and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed.

EXAMPLE QUESTIONS FROM SAMs: 1F q7a, q14a, q21a; 2F q9abf, 19abc

11. Expressions, formulae and rearranging equations**Teaching time**

5-7 hours

OBJECTIVES

2.2A	evaluate expressions by substituting numerical values for letters
2.3A	understand that a letter may represent an unknown number or a variable
2.3B	use correct notational conventions for algebraic expressions and formulae
2.3C	substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae
2.3D	use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols
2.3E	derive a formula or expression
2.3F	change the subject of a formula where the subject appears once

POSSIBLE SUCCESS CRITERIA

Evaluate the expressions for different values of x : $3x^2 + 4$ or $2x^3$.

There are 6 eggs in a small box and 12 eggs in a large box. Gary buys s small boxes and g large boxes. Write down an expression for the total number of eggs Gary buys.

Make t the subject of $v = u + at$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.

COMMON MISCONCEPTIONS

If $a = 2$ sometimes students interpret $3a$ as 32.

Making mistakes with negatives, including the squaring of negative numbers.

NOTES

Provide students with lots of practice.

This topic lends itself to regular reinforcement through starters in lessons.

Use formulae from mathematics and other subjects, expressed initially in words and then using letters and symbols.

EXAMPLE QUESTIONS FROM SAMs: 1F q8; 2F q9de

13. Sequences**Teaching time**

4-6 hours

OBJECTIVES

3.1A	generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
3.1B	find subsequent terms of an integer sequence and the rule for generating it
3.1C	use linear expressions to describe the n th term of arithmetic sequences

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'Which is the 1st term greater than 50?'

What is the amount of money after x months saving the same amount or the height of tree that grows 6 m per year?

What are the next terms in the following sequences?

1, 3, 9, ... 100, 50, 25, ... 2, 4, 8, 16, ...

Write down an expression for the n th term of the arithmetic sequence 2, 5, 8, 11, ...

Is 67 a term in the sequence 4, 7, 10, 13, ...?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.

COMMON MISCONCEPTIONS

The n th term of the sequence 1, 4, 7, 10 ... is $n + 3$ (rather than $3n - 2$)

NOTES

Emphasise use of $3n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the n th term.

Students are not expected to find the n th term of a quadratic sequence.

EXAMPLE QUESTIONS FROM SAMs: 1F q4; 2F q17

14. Real-life graphs**Teaching time**

3-5 hours

OBJECTIVES

3.3A	interpret information presented in a range of linear and non-linear graphs
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POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
 Read information from a distance–time or speed–time graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to draw a correct graph.
 Conversion graphs can be used to provide opportunities for students to justify which distance is further, or whether or not certain items can be purchase in different currencies.

COMMON MISCONCEPTIONS

With distance–time graphs, students struggle to understand that the perpendicular distance from the x -axis represents distance.

NOTES

Clear presentation of axes is important.
 Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
 Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.
 Use standard units of measurement to draw conversion graphs.
 Use various measures in distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.

EXAMPLE QUESTIONS FROM SAMs: 2F q5

16. Quadratic equations and graphs**Teaching time**

4–6 hours

OBJECTIVES

2.2E	expand the product of two simple linear expressions
2.2F	understand the concept of a quadratic expression and be able to factorise such expressions (limited to $x^2 + bx + c$)
2.7A	solve quadratic equations by factorization (limited to $x^2 + bx + c = 0$)
3.3I	recognise, generate points and plot graphs quadratic functions

POSSIBLE SUCCESS CRITERIASolve $3x^2 + 4 = 100$ Expand $(x + 2)(x + 6)$ Factorise $x^2 + 7x + 10$ Solve $x^2 + 7x + 10 = 0$ Solve $(x - 3)(x + 4) = 0$

Recognise a linear graph from its shape.

Recognise a quadratic graph from its shape.

Draw the graph of $y = x^2 + 3x - 4$ **OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Visual proof of the difference of two squares.

Given the length and width of a rectangle as expressions in x and the area of the rectangle, form a quadratic equation.**COMMON MISCONCEPTIONS** x terms can sometimes be 'collected' with x^2 .

Squaring negative numbers can be a problem.

NOTESEmphasise the fact that x^2 and x are different 'types' of term – illustrate this with numbers.

The graphs should be drawn freehand and in pencil, joining points using a smooth curve.

Encourage efficient use of the calculator.

EXAMPLE QUESTIONS FROM SAMs: 1F q21b

17. Simultaneous Equations**Teaching time**

3–5 hours

OBJECTIVES

2.6A	calculate the exact solution of two simultaneous equations in two unknowns
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POSSIBLE SUCCESS CRITERIA

Solve two simultaneous equations in two variables (linear/linear) algebraically

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Simple simultaneous equations can be formed and solved from real life scenarios, such as 2 adult and 2 child tickets cost £18, and 1 adult and 3 child tickets costs £17. What is the cost of 1 adult ticket?

COMMON MISCONCEPTIONS

The values of variables must be integer.

NOTES

Emphasise the need for good algebraic notation.
Clear algebraic working must be shown.

EXAMPLE QUESTIONS FROM SAMs: 2F q24

20. Polygons**Teaching time**

4-6 hours

OBJECTIVES

4.2A	recognise and give the names of polygons
4.2D	understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons
4.2E	understand and use the angle sum of polygons

POSSIBLE SUCCESS CRITERIA

Deduce and use the angle sum in any polygon.

Derive the angle properties of regular polygons.

Given the size of its exterior angle, how many sides does the polygon have?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems whereby students have to justify the number of sides that a regular polygon has given an interior or exterior angle.

COMMON MISCONCEPTIONS

Pupils may believe, incorrectly, that all polygons are regular.

NOTES

Study Escher drawings.

Use examples of tiling patterns with simple shapes to help students investigate if shapes 'fit together'.

EXAMPLE QUESTIONS FROM SAMs: 2F q26

21. Compound Measure**Teaching time**

4-6 hours

OBJECTIVES

4.4F	understand and use the relationship between average speed, distance and time
4.4G	use compound measure such as speed, density and pressure

POSSIBLE SUCCESS CRITERIA

Find the speed given distance and time.

Find the distance (in km) given the speed (in km/h) and the time (in minutes).

Recall and use the formula for density.

Give the formula for pressure, use it to find one of the variables.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the mass of an object, having first to find its volume.

Work out the average speed of a journey.

COMMON MISCONCEPTIONS

Using inconsistent units when solving problems.

Converting time into a decimal incorrectly. E.g. writing 1 hour 15 minutes as 1.15 hours.

NOTES

Practice converting time into decimals.

Ensure that conversions between metric units are known.

EXAMPLE QUESTIONS FROM SAMs: 1F q16; 2F q18

23. Circles and cylinders**Teaching time**

5-7 hours

OBJECTIVES

4.6A	recognise the terms 'centre', 'radius', 'chord', 'diameter', 'circumference', 'tangent', 'arc', 'sector' and 'segment' of a circle
4.6B	understand chord and tangent properties of circles
4.9E	find circumferences and areas of circles using relevant formulae; find perimeters and areas of semicircles
4.10D	find the surface area of a cylinder
4.10E	find the volume of prisms, including cuboids and cylinders, using an appropriate formula

POSSIBLE SUCCESS CRITERIA

Recall terms related to a circle.

Understand that answers in terms of pi are more accurate.

Find the volume of a cylinder given the height and diameter.

Find the area and circumference of a circle given the radius or diameter.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Calculate the radius/diameter given the area/circumference type questions could be explored, including questions that require evaluation of statements, such as Andy states "Diameter = $2 \times$ Radius" and Bob states "Radius = $2 \times$ Diameter". Who is correct?

Problems involving straight-forward and compound shapes in a real-life context should be explored to reinforce the concept of area. For example, the floor plan of a room linked to the amount of flooring needed.

Problems using number of revolutions of a wheel.

COMMON MISCONCEPTIONS

Diameter and radius are often confused and recollection which formula to use for area and circumference of circles is often poor.

Volume often gets confused with surface area.

NOTES

Emphasise the need to learn the circle formula: 'Cherry Pie's Delicious' and 'Apple Pies are too' are good ways to remember them.

Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

EXAMPLE QUESTIONS FROM SAMs: 1F q25

25. Pythagoras' Theorem and Trigonometry**Teaching time**

11-13 hours

OBJECTIVES

4.8A	know, understand and use Pythagoras' Theorem in two dimensions
4.8B	know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle
4.8C	apply trigonometrical methods to solve problems in two dimensions

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

Find a given side or angle using trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65° . What is the greatest height it can reach?

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer or round their answer prematurely.

NOTES

Students may need reminding about surds.

Drawing the squares on the 3 sides will help to illustrate the theorem.

Include examples with triangles drawn in all four quadrants.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.

EXAMPLE QUESTIONS FROM SAMs: 1F q22

26. Similarity and congruence in 2D**Teaching time**

4-6 hours

OBJECTIVES

4.2F	understand congruence as meaning the same shape and size
4.2G	understand that two or more polygons with the same shape and size are said to be congruent to each other
4.11A	understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged

POSSIBLE SUCCESS CRITERIA

Understand similarity as one shape being an enlargement of the other.

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using scale diagrams, including bearings and maps, provides a rich source of real-life examples and links to other areas of mathematics.

COMMON MISCONCEPTIONS

Students may incorrectly believe that all polygons are regular or that all triangles have a rotational symmetry of order 3.

Often students think that when a shape is enlarged the angles also get bigger.

NOTES

Use simple scale factors that are easily calculated mentally to introduce similar shapes.

Reinforce the fact that the sizes of angles are maintained when a shape is enlarged.

EXAMPLE QUESTIONS FROM SAMs: 2F q6

27. Constructions and bearings**Teaching time**

3-5 hours

OBJECTIVES

4.5B	construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses
4.5D	use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle

POSSIBLE SUCCESS CRITERIA

Construct a given triangle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link problems with other areas of mathematics, such as the trigonometric ratios and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Construction arcs should be left in.

EXAMPLE QUESTIONS FROM SAMs: 1F q20; 2F q6

28. Graphical representation of data**Teaching time**

6-8 hours

OBJECTIVES

6.1A	use different methods of presenting data
6.1B	use appropriate methods of tabulation to enable the construction of statistical diagrams
6.1C	interpret statistical diagrams

POSSIBLE SUCCESS CRITERIA

Construct a frequency table.

Interpret and draw a pictogram.

Interpret and draw a bar chart.

From a simple pie chart identify the frequency represented by $\frac{1}{4}$ and $\frac{1}{2}$ sections.

Find the angle for one item.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to present information.

From inspection of a pie chart, students should be able to identify the fraction of the total represented and know when that total can be calculated and compared with another pie chart

COMMON MISCONCEPTIONS

Students struggle to make the link between what the data in a frequency table represents, so for example may state the 'frequency' rather than the interval when asked for the modal group.

In a pie chart, same size sectors for different sized data sets represent the same number rather than the same proportion.

NOTES

Ensure that you include a variety of scales, including decimal numbers of millions and thousands, time scales in hours, minutes, seconds.

Relate $\frac{1}{4}$, $\frac{1}{2}$, etc to percentages.

Practise dividing by 20, 30, 40, 60, etc.

Compare pie charts to identify similarities and differences.

Angles when drawing pie charts should be accurate to 2°.

EXAMPLE QUESTIONS FROM SAMs: 1F q3; 2F q11

29. Statistical measures**Teaching time**

6-8 hours

OBJECTIVES

6.2A	understand the concept of average
6.2B	calculate the mean, median, mode and range for a discrete data set
6.2C	calculate an estimate for the mean for grouped data
6.2D	identify the modal class for grouped data

POSSIBLE SUCCESS CRITERIA

State the median, mode, mean and range from a small data set.

Estimate the mean from a table of grouped and from a table of ungrouped data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide a correct solution as a counter-argument to statements involving the “averages”, e.g. Susan states that the median is 15, she is wrong. Explain why.

Given the mean, median and mode of five positive whole numbers, can you find the numbers?

COMMON MISCONCEPTIONS

Often the $\Sigma(m \times f)$ is divided by the number of classes rather than Σf when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f .

Remind students how to find the midpoint of two numbers.

EXAMPLE QUESTIONS FROM SAMs: 1F q18; 2F q22

International GCSE Mathematics
(Specification A)
(4MA1)
Higher Tier

Scheme of Work

Unit number		Title	Estimated teaching hours
Number	1	Decimals	4
	2	Special numbers, powers and roots	6
	3	Fractions	4
	4	Percentages	5
	5	Ratio and proportion	3
	6	Indices and standard form	4
	7	Degree of accuracy	4
	8	Set language, notation and Venn diagrams	6
Algebra	9	Algebraic manipulation	8
	10	Expressions, formulae and rearranging formulae	6
	11	Linear equations and inequalities	4
	12	Sequences	4
	13	Real life graphs	2
	14	Linear graphs	7
	15	Quadratic equations and graphs	8
	16	Harder graphs and transformation of graphs	7
	17	Simultaneous equations	5
	18	Function notation	7
	19	Calculus	8
Space, shape and measure	20	Compound measures	5
	21	Geometry of shapes	6
	22	Constructions and bearings	4
	23	Perimeter, area and volume	8
	24	Pythagoras' theorem and Trigonometry	8
	25	Transformations	5
	26	Circle properties	6
	27	Advanced trigonometry	8
	28	Similar shapes	7
	29	Vectors	6
Handling Data	30	Graphical representation of data	5
	31	Statistical measures	4
	32	Probability	6
		Total	180

12	Sequences	3.1A	understand and use common difference (d) and first term (a) in an arithmetic sequence	4
		3.1B	know and use n th term $= a + (n - 1)d$	
		3.1C	find the sum of the first n terms of an arithmetic series (S_n)	
13	Real life graphs	3.3A	interpret information presented in a range of linear and non-linear graphs	2
14	Linear graphs	3.3E	determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points	7
		3.3G	find the gradient of a straight line	
		3.3H	recognize that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$	
		3.3I	recognize, generate points and plot graphs of linear functions	
		3.3F	calculate the gradient of a straight line given the coordinates of two points	
		3.3G	find the equation of a straight line parallel to a given line; find the equation of a straight line perpendicular to a given line	
		2.8D	represent simple linear inequalities on rectangular Cartesian graphs	
		2.8E	identify regions on rectangular Cartesian graphs defined by simple linear inequalities	
15	Quadratic equations, inequalities and graphs	2.8B	identify harder examples of regions defined by linear inequalities	8
		2.7A	solve quadratic equations by factorization	
		2.7B	solve quadratic equations by using the quadratic formula or completing the square	
		2.7C	form and solve quadratic equations from data given in a context	
		2.8A	solve quadratic inequalities in one unknown and represent the solution set on a number line	
		3.3I	recognize, generate points and plot graphs of quadratic functions	

Higher tier

		3.4E apply calculus to linear kinematics and to other simple practical problems	
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		<p>4.6C understand and use angle properties of the circle including:</p> <ul style="list-style-type: none"> (i) angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the remaining part of the circumference (ii) angle subtended at the circumference by a diameter is a right angle (iii) angles in the same segment are equal (iv) the sum of the opposite angles of a cyclic quadrilateral is 180° (v) the alternate segment theorem 	
27	Advanced trigonometry	4.8C understand and use the sine and cosine rules for any triangle	8
		4.8D use Pythagoras' theorem in three dimensions	
		4.8E understand and use the formula $\frac{1}{2}ab \sin C$ for the area of a triangle	
		4.8F apply trigonometrical methods to solve problems in three dimensions, including finding the angle between a line and a plane	
28	Similar shapes	4.2F understand congruence as meaning the same shape and size	7
		4.2G understand that two or more polygons with the same shape and size are said to be congruent to each other	
		4.11A understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged	
		4.11A understand that areas of similar figures are in the ratio of the square of corresponding sides	
		4.11B understand that volumes of similar figures are in the ratio of the cube of corresponding sides	
		4.11C use areas and volumes of similar figures in solving problems	
29	Vectors	5.1A understand that a vector has both magnitude and direction	6
		5.1B understand and use vector notation including column vectors	
		5.1C multiply vectors by scalar quantities	
		5.1D add and subtract vectors	
		5.1E calculate the modulus (magnitude) of a vector	
		5.1F find the resultant of two or more vectors	
		5.1G apply vector methods for simple geometrical proofs	

It is assumed that students being prepared for the Higher tier will have knowledge of the Foundation tier content.

1. Decimals

Teaching time
3-5 hours

OBJECTIVES

H1.3A	convert recurring decimals into fractions
F1.8B	round to a given number of significant figures or decimal places
F1.8D	use estimation to evaluate approximations to numerical calculations
F1.11A	use a scientific electronic calculator to determine numerical results

POSSIBLE SUCCESS CRITERIA

Estimate the value of $\frac{34.5 \times 7.34}{0.154}$

Change $0.4\dot{5}$ into a fraction in its simplest form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of decimals within a problem.

Show algebraically that $3.0\dot{1}$ can be written as $3\frac{1}{90}$

Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some pupils may think $35\,934 = 36$ to two significant figures.

NOTES

The expectation for Higher tier is that much of this work will be reinforced throughout the course.

Make sure students are absolutely clear about the difference between significant figures and decimal places.

EXAMPLE QUESTIONS FROM SAMs: -

2. Special numbers and powers**Teaching time**

5-7 hours

OBJECTIVES

F1.4D	express integers as product of powers of prime factors
F1.4E	find highest common factors (HCF) and lowest common multiples (LCM)
H1.4A	understand the meaning of surds
H1.4B	manipulate surds, including rationalising a denominator
H1.4C	use index laws to simplify and evaluate numerical expressions involving integer, fractional and negative powers

POSSIBLE SUCCESS CRITERIA

What is the value of 2^5 ?

Find the HCF and LCM of 12 and 20

Write a number as a product of its prime factors.

Prove that the square root of 45 lies between 6 and 7.

Simplify $\sqrt{40}$

Rationalise the denominator of $\frac{5}{\sqrt{10}}$; $\frac{6}{1+\sqrt{2}}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.

Use negative number and not minus number to avoid confusion with calculations.

Students need to be encouraged to learn squares from 2×2 to 15×15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.

EXAMPLE QUESTIONS FROM SAMs: 4H q1, q24

6. Indices and standard form**Teaching time**

3-5 hours

OBJECTIVES

F1.4C	use index notation and index laws for multiplication and division of positive and negative integer powers including zero
F1.9A	calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$
H1.9A	solve problems involving standard form

POSSIBLE SUCCESS CRITERIA

Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0 , $8^{\frac{2}{3}}$.

Work out the value of n in $40 = 5 \times 2^n$.

Write 51080 in standard form.

Write 3.74×10^{-6} as an ordinary number.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.

NOTES

Standard form is used in science and there are lots of cross-curricular opportunities. Students need to be provided with plenty of practice in using standard form with calculators.

EXAMPLE QUESTIONS FROM SAMs: 3H q9, 4H 4d

7. Degree of accuracy**Teaching time**

3-5 hours

OBJECTIVES

F1.8C	identify upper and lower bounds where values are given to a degree of accuracy
H1.8A	solve problems using upper and lower bounds where values are given to a degree of accuracy

POSSIBLE SUCCESS CRITERIA

Round 16,000 people to the nearest 1000.

Round 1100 g to 1 significant figure.

Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.

Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use 'half a unit above' and 'half a unit below' to find upper and lower bounds. Encourage use a number line when introducing the concept.

EXAMPLE QUESTIONS FROM SAMs: 3H q17

12. Sequences**Teaching time**

3-5 hours

OBJECTIVES

H3.1A	understand and use common difference (d) and first term (a) in an arithmetic sequence
H3.1B	know and use n th term $= a + (n - 1)d$
H3.1C	find the sum of the first n terms of an arithmetic series (S_n)

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'which is the 1st term greater than 50?'

Given the sequence 12, 7, 2, -3... find an expression in terms of n for the n th term.

Be able to solve problems involving sequences from real-life situations, such as:

- What is the amount of money after x months saving the same amount, or the height of tree that grows 6 m per year;

Given the sequence 5, 8, 11, 14... find the 50th term, the sum of the first 50 terms

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to " n ".

Writing $n + 3$ instead of $3n - 1$ for the n th term of 2, 5, 8, 11...

NOTES

Emphasise use of $3n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the n th term.

EXAMPLE QUESTIONS FROM SAMs: 3H q23; 4H q2

13. Real-life graphs**Teaching time**

3-4 hours

OBJECTIVES

F3.3A	interpret information presented in a range of linear and non-linear graphs
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POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.

COMMON MISCONCEPTIONS

Reading scales incorrectly is a common cause of errors.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.

Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.

Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.

EXAMPLE QUESTIONS FROM SAMs: -

17. Simultaneous equations**Teaching time**

4-6 hours

OBJECTIVES

H2.6A	calculate the exact solution of two simultaneous equations in two unknowns
H2.6B	interpret the equations as lines and the common solution as the point of intersection
H2.7D	solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic

POSSIBLE SUCCESS CRITERIA

Solve the simultaneous equations $2x + 5y = -14$; $3x - 4y = 25$

Solve the simultaneous equations $x^2 + y^2 = 18$; $2x + 1 = y$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

Link the solution of simultaneous equations to their graphical representation.

COMMON MISCONCEPTIONS

Some students always discard solutions with negative values.

NOTES

Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.

Clear presentation of working out is essential.

Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: 4H q9

18. Function notation**Teaching time**

6-8 hours

OBJECTIVES

H3.2A	understand the concept that a function is a mapping between elements of two sets
H3.2B	use function notations of the form $f(x) = \dots$ and $f : x \mapsto \dots$
H3.2C	understand the terms 'domain' and 'range' and which values may need to be excluded from a domain
H3.2D	understand and find the composite function fg and the inverse function f^{-1}

POSSIBLE SUCCESS CRITERIA

Given $f(x) = 3 - 5x$; find $f(2)$, $f^{-1}(3)$

Given $g(x) = \frac{2}{3-x}$, write down the value of x that must be omitted from any domain of g .

Find $fg(4)$; $gf(4)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations using functions. E.g. solve $f(x) = g(x)$

Give the graph of $f(x)$ and use that to find $f(3)$ and $f(x) = 2$

COMMON MISCONCEPTIONS

Confusing $gf(x)$ with $fg(x)$

NOTES

Link with algebraic manipulation and equation solving.

EXAMPLE QUESTIONS FROM SAMs: 4H q17

19. Calculus**Teaching time**

7-9 hours

OBJECTIVES

H3.4A	understand the concept of a variable rate of change
H3.4B	differentiate integer powers of x
H3.4C	determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs
H3.4D	distinguish between maxima and minima by considering the general shape of the graph only
H3.4E	apply calculus to linear kinematics and to other simple practical problems

POSSIBLE SUCCESS CRITERIA

Differentiate $8x^3 + 3x + 2$; $\frac{2}{x^2} + 3x$

Find the turning point of $y = x^2 + 8x - 20$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the values of x for which the graph of $y = x^2 - x + 3$ has a gradient of 7

Given that $s = t^3 + 2t^2$ find the value of t for which the particle is instantaneously at rest.

COMMON MISCONCEPTIONS

3 differentiates to 3 (rather than 0)

NOTES

Link with solving linear and quadratic equations.

EXAMPLE QUESTIONS FROM SAMs: 3H q21; 4H q25

20. Compound measures**Teaching time**

4-6 hours

OBJECTIVES

F4.4G	use compound measure such as speed, density and pressure
F4.9A	convert measurements within the metric system to include linear and area units
F4.10A	convert between units of volume within the metric system

POSSIBLE SUCCESS CRITERIA

Find the speed given distance and time.

Find the distance (in km) given the speed (in km/h) and the time (in minutes).

Recall and use the formula for density.

Give the formula for pressure, use it to find one of the variables.

Change 4 m^2 into cm^2 .

Change 45 mm^2 into cm^2 .

Change 3000 cm^3 into m^3 .

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the mass of an object, having first to find its volume.

Work out the average speed of a journey.

COMMON MISCONCEPTIONS

Using inconsistent units when solving problems.

Converting time into a decimal incorrectly. E.g. writing 1 hour 15 minutes as 1.15 hours.

NOTES

Practice converting time into decimals.

Ensure that conversions between metric units are known.

Ensure that consistent units are used when solving problems.

EXAMPLE QUESTIONS FROM SAMs: 3H q1; 4H q3

22. Constructions and bearings**Teaching time**

3-5 hours

OBJECTIVES

F4.5B	construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses
F4.5D	use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle
F4.4D	understand angle measure including three-figure bearings
F4.5C	solve problems using scale drawings
F4.11B	use and interpret maps and scale drawings

POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.

When given the bearing of a point A from point B , can work out the bearing of B from A .

Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.

Construct the perpendicular bisector of a given angle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving combinations of bearings and scale drawing can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.

Construction lines should not be erased.

EXAMPLE QUESTIONS FROM SAMs: 3H q5

24. Pythagoras' Theorem and trigonometry**Teaching time**

7-9 hours

OBJECTIVES

F4.8A	know, understand and use Pythagoras' Theorem in two dimensions
F4.8B	know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle
F4.8C	apply trigonometrical methods to solve problems in two dimensions
H4.8A	understand and use sine, cosine and tangent of obtuse angles
H4.8B	understand and use angles of elevation and depression

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

Link to 'real-life' situations E.g. link with bearings and scale drawings.

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.

EXAMPLE QUESTIONS FROM SAMs: 3H q7

26. Circle theorems**Teaching time**

5-7 hours

OBJECTIVES

H4.6A	understand and use the internal and external intersecting chord properties
H4.6B	recognise the term 'cyclic quadrilateral'
H4.6C	understand and use angle properties of the circle including: (i) angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the remaining part of the circumference (ii) angle subtended at the circumference by a diameter is a right angle (iii) angles in the same segment are equal (iv) the sum of the opposite angles of a cyclic quadrilateral is 180° (v) the alternate segment theorem

POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems, giving a reason for each stage in working.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counter-arguments to statements.

Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.
There is often confusion when identifying cyclic quadrilaterals.

NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout. Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

EXAMPLE QUESTIONS FROM SAMs: 3H q16

27. Advanced trigonometry**Teaching time**

7-9 hours

OBJECTIVES

H4.8C	understand and use the sine and cosine rules for any triangle
H4.8D	use Pythagoras' theorem in three dimensions
H4.8E	understand and use the formula $\frac{1}{2}ab\sin C$ for the area of a triangle
H4.8F	apply trigonometrical methods to solve problems in three dimensions, including finding the angle between a line and a plane

POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.

Justify when to use the cosine rule, sine rule, Pythagoras' Theorem or normal trigonometric ratios to solve problems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semi-circle can provide links with other areas of mathematics.

Multi-step problems requiring the use of both the Sine rule and Cosine rule.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.

When finding angles students will often be unable to rearrange the cosine rule or fail to find the inverse of $\cos \theta$.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.

Ensure that finding angles with 'normal trig' is refreshed prior to this topic.

Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.

In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

EXAMPLE QUESTIONS FROM SAMs: 3H q19; 4H q21

28. Similar shapes**Teaching time**

6-8 hours

OBJECTIVES

F4.2F	understand congruence as meaning the same shape and size
F4.2G	understand that two or more polygons with the same shape and size are said to be congruent to each other
F4.11A	understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged
H4.11A	understand that areas of similar figures are in the ratio of the square of corresponding sides
H4.11B	understand that volumes of similar figures are in the ratio of the cube of corresponding sides
H4.11C	use areas and volumes of similar figures in solving problems

POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.

Understand that enlargement does not have the same effect on area and volume.

Given the volumes of two similar shapes and the surface area of one, find the surface area of the other shape.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

NOTES

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.

Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.

EXAMPLE QUESTIONS FROM SAMs: 4H q18

29. Vectors**Teaching time**

5-7 hours

OBJECTIVES

H5.1A	understand that a vector has both magnitude and direction
H5.1B	understand and use vector notation including column vectors
H5.1C	multiply vectors by scalar quantities
H5.1D	add and subtract vectors
H5.1E	calculate the modulus (magnitude) of a vector
H5.1F	find the resultant of two or more vectors
H5.1G	apply vector methods for simple geometrical proofs

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.

Solve geometric problems and produce proofs.

Find the magnitude of a vector.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

"Show that"-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics, in particular algebra.

Find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.

Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.

Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.

Extend geometric proofs by showing that the medians of a triangle intersect at a single point.

EXAMPLE QUESTIONS FROM SAMs: 3H q22; 4H q23

30. Graphical representation of data**Teaching time**

4-6 hours

OBJECTIVES

H6.1A	construct and interpret histograms
H6.1B	construct cumulative frequency diagrams from tabulated data
H6.1C	use cumulative frequency diagrams

POSSIBLE SUCCESS CRITERIA

Construct cumulative frequency graphs and histograms from frequency tables.

Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from cumulative frequency graphs and relate the key measures in the context of the data.

COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' or 'Cumulative Frequency'.

Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

Histograms are often not well understood with the height used for frequency rather than the area.

NOTES

Ensure that axes are clearly labelled.

EXAMPLE QUESTIONS FROM SAMs: 3H q14; 4H q12

OBJECTIVES

F6.3C	understand and use estimates or measures of probability from theoretical models
F6.3D	find probabilities from a Venn diagram
F6.3E	understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space
F6.3G	estimate probabilities from previously collected data
F6.3H	calculate the probability of the complement of an event happening
F6.3I	use the addition rule of probability for mutually exclusive events
F6.3J	understand and use the term 'expected frequency'
H6.3A	draw and use tree diagrams

POSSIBLE SUCCESS CRITERIA

If the probability of outcomes are x , $2x$, $4x$, $3x$, calculate x .

Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

Use a tree diagram to find the probability of a combined event.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.

Not using fractions or decimals when working with probability trees.

NOTES

Encourage students to work 'across' the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1.

If a question says, for example, that 'two counters are taken from a bag' then, by implication, this is a non-replacement probability question.

EXAMPLE QUESTIONS FROM SAMs: 3H q3c; 4H q15bc, q20

